HEG-003-1161002

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Seat No. ________

M. Sc. (Mathematics) (Sem. I) (CBCS) Examination

November / December – 2017

Real Analysis : MATH CMT-1002

(New Course)

Faculty Code : 003
Subject Code : 1161002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Answer all questions.

(2) Each question carries 14 marks.

(3) The figures to the right indicate marks allotted to the question.

1 All are compulsory (Each question carries **two** marks) **14**

(a) Define algebra of sets.

(b) Give an example of a set that is σ algebra of sets.

(c) Give an example of a F_σ – set.

(d) True or false : $\mathbb{Q}$, the set of rationals, is a G_δ – set.

(e) Define measurable function.

(f) State Littelwood's third principle.

(g) Define function of bounded variation.

2 Answer any **two** : **14**

(a) Prove that every closed and open set are measurable. **7**

(b) Define Lebesgue outer measure of a set and show that Lebesgue outer measure of a finite interval is its length. **7**

(c) Show that countable union of measurable sets is again measurable. **7**

3 All are compulsory : **14**

(a) Prove that if f and g are measurable functions then fg is also measurable. **7**

(b) Show that f is a function of bounded variation on [a, b] if and only if there exists monotonically increasing functions g, h: [a, b] $\to \mathbb{R}$ such that $f = g - h$. **7**

OR

3 All are compulsory : **14**

(a) State and prove Egoroff's theorem. **7**

(b) State and prove Fatou's Lemma. **7**

4 Answer any **two** : **14**

(a) State and prove Lebesgue dominated convergence theorem. **7**

(b) Define lebesgue integral of a bounded measurable function. If f and g are bounded measurable functions defined on measurable set E then show that **7**

$\int_E af + bg = a\int_E f + b\int_E g.$

(c) State and prove Bounded convergence theorem. **7**

5 All are compulsory (each question carries **two** marks) **14**

(a) Show that if E is measurable set then its complement is also measurable.

(b) Show that [a, b] is uncountable.

(c) Give the Lebesgue outer measure of a countable subset of $\mathbb{R}$.

(d) Let $\langle f_n \rangle$ be a sequence of measurable functions defined on E. If f: E $\to \mathbb{R}$ then when do we say that $\langle f_n \rangle$ converges to f in measure.

(e) Show that every step function is measurable.

(f) State monotone convergence theorem.

(g) True false : Fatou's Lemma and Lebesgue dominated convergence theorem holds good if almost every where is replaced by convergence in measure ?